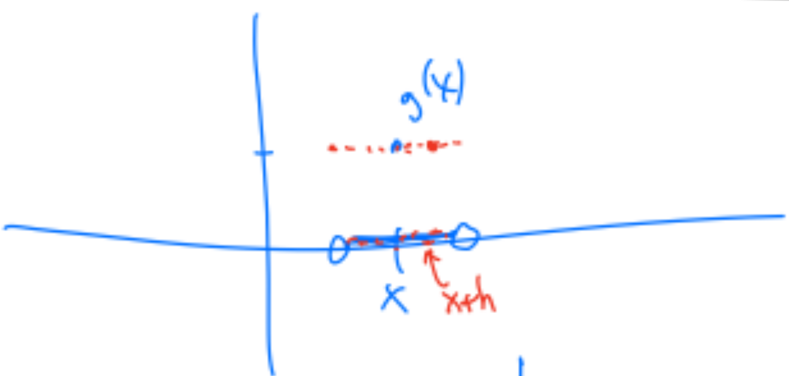
Examples

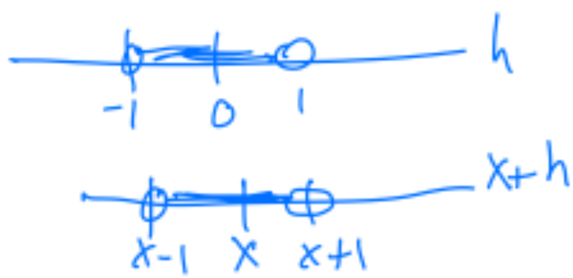
- Find $\lim\limits_{\theta \to 0} \dfrac{\sin^5\theta}{\theta^8 - 3\theta^6 - 6\theta^5}$

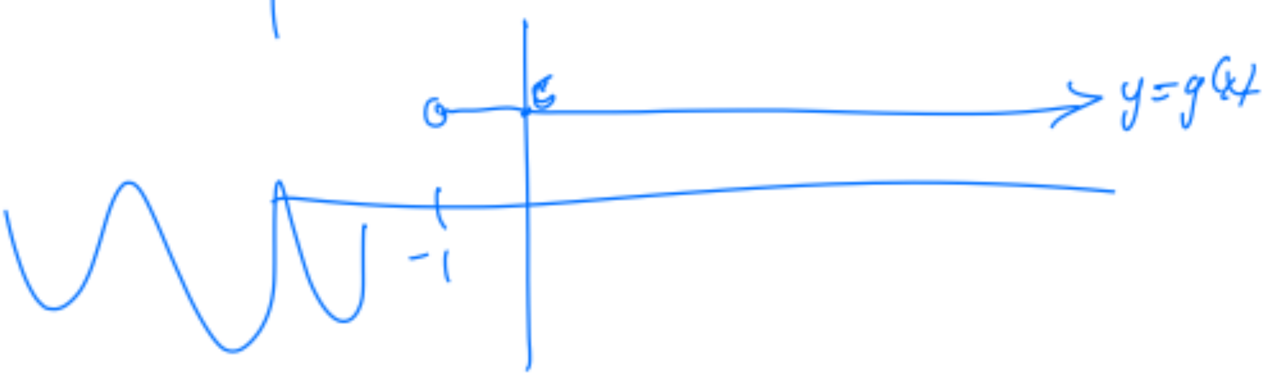
Solution: limit $= \lim\limits_{\theta\to 0} \dfrac{\sin^5\theta}{\theta^5(\theta^3 - 3\theta - 6)}$

$$= \lim_{\theta\to 0} \underbrace{\left(\frac{\sin\theta}{\theta}\right)^5}_{\to 1} \cdot \underbrace{\frac{1}{\theta^3 - 3\theta - 6}}_{\to -6} = \boxed{\frac{-1}{6}}$$

(1.15) Suppose a fcn g satisfies $g(x+h) = g(x)$ for all x positive and for all h such that $|h| < 1$. What does this tell us about g in simple terms?







Special Limits

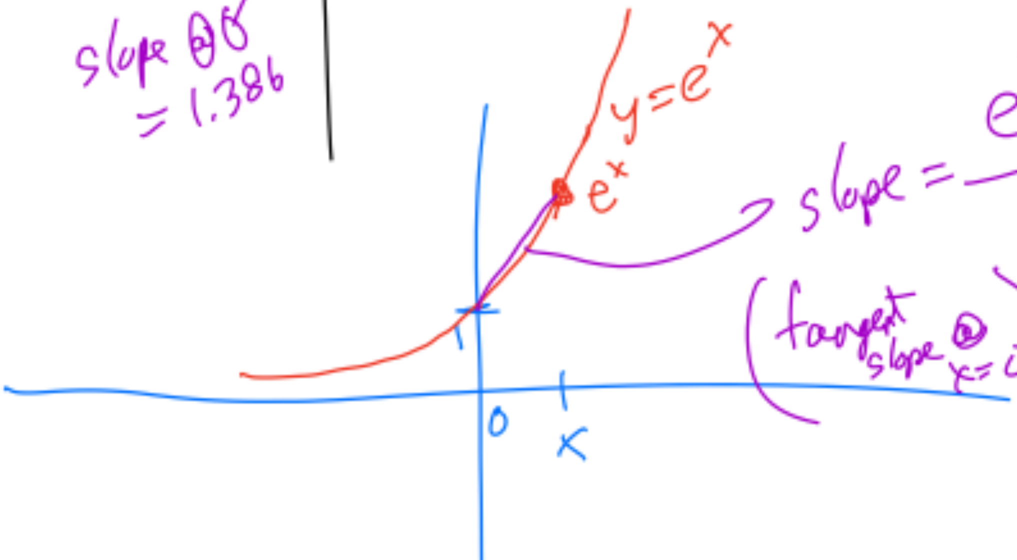
① $$\lim_{\theta \to 0} \frac{\sin\theta}{\theta} = 1$$

② $$\lim_{x \to 0} \frac{e^x - 1}{x} = 1$$

(One definition of e).

instantaneous rate of change = tangent slope of $y = e^x$ at $x = 0$.

Calc. $e \approx 2.71828\ldots$



$$\text{slope} = \frac{e^x - 1}{x - 0} = \frac{e^x - 1}{x}$$

$$(\text{tangent slope @ } x=0) = \lim_{x \to 0} \frac{e^x - 1}{x} = 1$$

③ (Same as #2 after messing around with algebra & logs)

$$\lim_{t \to \infty} \left(1 + \frac{1}{t}\right)^t = e$$

(you also take this as the definition of e)

Examples

(0) $$\lim_{x\to 0} \underbrace{\frac{\sin(x^5)}{x^5}}_{\to 1} + \underbrace{\frac{e^{2x^7}-1}{2x^7}}_{1} = 1+1=2.$$

$x^5 \to 0$

(1) $$\lim_{p\to 0} \frac{\sin^5(2p)}{e^{p^5}-1} = \lim_{p\to 0} \frac{\left(\frac{\sin(2p)}{2p}\right)^5 \cdot (2p)^5}{\left(\frac{e^{p^5}-1}{p^5}\right)\cdot p^5}$$

$\frac{\sin(2p)}{2p} \to 1$, $\frac{e^{p^5}-1}{p^5} \to 1$

$$= \lim_{p\to 0} \frac{(2p)^5}{p^5} = \lim_{p\to 0} \frac{2^5 \cancel{p^5}}{\cancel{p^5}} = 2^5 = \boxed{32}$$

(2) $$\lim_{h\to 0} \frac{3^h-1}{4h} = \lim_{h\to 0} \frac{e^{(\ln 3)h}-1}{4h}$$

$$\lim_{x\to 0} \frac{e^x-1}{x}$$

$$3^h = e^x$$

$$\ln(3^h) = \ln(e^x) = x$$

$$h(\ln 3) = x$$

$$e^{\ln 3} = 3$$

$$3^h = \left[e^{(\ln 3)}\right]^h = e^{(\ln 3)h}$$

$$= \lim_{h\to 0} \frac{e^{(\ln 3)h} - 1}{(\ln 3)h \left(\frac{4}{\ln 3}\right)} = \lim_{h\to 0} \frac{e^{(\ln 3)h} - 1}{(\ln 3)h} \cdot \left(\frac{\ln 3}{4}\right)$$

$$\frac{e^{(\ln 3)h} - 1}{(\ln 3)h} \to 1$$

$$= \boxed{\frac{\ln 3}{4}}$$